

A Construction Method of Digital Screen Sets

Realization of Moire-free Rational Tangent Screens by Using the Multi-unit Area Design Method

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Abstract

In many cases including the Adobe Accurate Screen Method[3], designing rational tangent screen is performed in the image domain. This paper reports a newly developed construction method of digital screen sets. For the proposed method, rational tangent screens having screen angles of ±15 degrees and 45 degrees are designed in the spatial frequency domain.

In Section 2.0, the characteristics of frequency components generated by superimposing the traditional ±15 degrees screens and the most appropriate combination of ±15 degrees screens and 45 degrees screen to avoid the production of undesirable moire pattern and to get the circular rosette shape are investigated.

In the study process, the rosette shapes for various rational tangent screens are studied precisely in the spatial frequency domain and it is analyzed that rational tangent values (dot alignment) of 3/11, 4/15 and 5/19 are most suitable for 15 degrees screens to produce circular rosettes and the size of the 45 degrees screen combined with each rational tangent screen can be calculated form the rational tangent values. The Section 4.0 concentrates on explaining of design process in the spatial frequency domain and the conversion between domains.

The frequency component array corresponding to the most suitable rational tangent value 3/11, 4/15 or 5/19 is converted to the Unit Area (UA) array in the image domain.

As the result of conversion, the minimum size of unit area array for halftoning called the Minimum Repetition Unit (MRU) is obtained and the MRU becomes the large scale supercell construction.

Examples of various design parameters are tabulated in Table 2.

Because the MRU contains lots of UAs in each of which a dot grows up and therefore, the design method proposed should require the mapping of threshold values.

The authors have named this method "the Multi-unit Area design Method (MAM)". For the mapping of threshold values to the UAs the new technology is required to simplify and to implement automatically the assignment of threshold values.

The essential part of this technology is explained in Section 5.0 and a halftone image to which the MAM is applied is depicted in Fig. 9.

In conclusion, the proposed digital halftone design method named "the Multi-unit Area Design Method" should serve a theoretical basis as moire-free and circular rosette digital screening.

Abbreviation

MAM:	Multi-unit Area design Method
RU:	Repetition Unit having a area size of $(\beta \cdot p \cdot q)^2$
MRU:	Minimum Repetition Unit having a area size of $(p \cdot q)^2$
RT:	Rational Tangent value
UA:	Unit Area in which a dot grows
MTP:	Minimum area of the Threshold Pattern to be stored in the memory
RPF:	Rosette Pattern in the spatial Frequency domain
RPI:	Rosette Pattern in the Image domain
SP:	Standard Pattern,
SPF:	Standard Pattern Function defined by $\mathbf{T} = k \cdot (\lvert x \rvert + \lvert y \rvert)/2$

1.0 Introduction

In process color printing, three (or four) halftone screens, one for each of the colors cyan, magenta and yellow (black) are superimposed. These screens having screen angles of ±15 degree and 45 degree constitute a screen set.

When halftone screens having rational tangent values [RTs]are superimposed upon one another, as happens in process color printing, the process usually produces an objectionable moire pattern that reduces the quality of the printed image.

So, the various methods to avoid the moire patterns are adopted in practical systems[1,2] To prevent the undesirable moire patterns, Peter Fink proposed the Supercell Method named the Adobe Accurate Screen to realize the screen angle having RT with the required accuracy.[3] The supercell designed in the image domain including m × m (m: integer) identical halftone cells is one solution to approximate accurately the required screen angle and screen ruling.

Moire pattern analysis in the spatial domain has been a long-term focus of our research. Recently we have developed the new method named the Multi-unit Area design Method (**MAM**) proposed in this paper.

The coordinates of the frequency spectrums around the origin for moire-free combinations of ±15 degrees screens having various RTs and the 45 degree screen are calculated and the shapes of the spectrum dispositions are compared in the spatial frequency domain. The results depicted show that the circular rosette patterns appear only when the RT is 3/11, 4/15 or 5/19 for 15 degrees halftone screens.

Next, the conversion method from frequency components array corresponding to the most suitable RTs for the circular rosette to the multi-unit area (cell) array in the image domain is discussed. The multi-unit area array converted inevitably must include a larger number of identical cells in the minimum repetition unit (**MRU**) than the Supercell by the Adobe Accurate Screen method.

In real systems, how to satisfy tonal values and how to make the dot shape are important issues. For this purpose, the scaling factor β was adopted and a computerized mapping method is developed and applied to real system design.

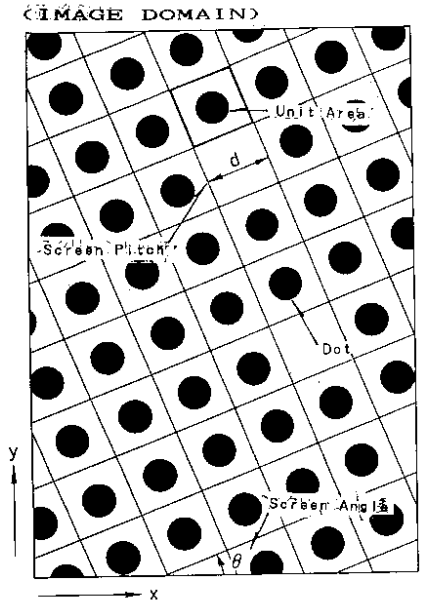
Printed example is depicted in Fig. 9. The circular rosette moire can be observed in this picture and quality of this halftone seems to be good enough for commercial prints.

2.0 Recognition of Traditional Screening Method

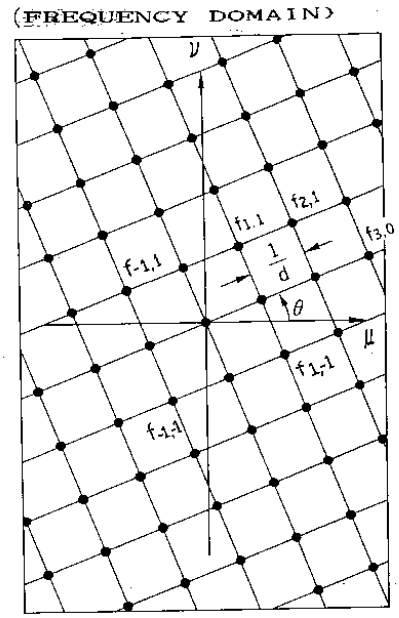
For half-tone dots of color without a preferential axis, the normal screening for three colors, cyan, magenta and yellow are usually 30 degrees from another color. The spatial frequency distribution of the overlapped image of two separation films can be analyzed in the frequency domain effectively.

Fig. 1 (a) shows the dot alignment of the halftone and Fig. 1 (b) shows the corresponding frequency components distribution. Each frequency component is nominated by $\mathbf{f}^{\theta}_{mn}$ having coordinates of [μ,ν] [mn: 0,±1,±2•, θ: screen angle] as described in the Fig.1 (b). The coordinates [μ,ν] are expressed by equation (1).

$$\mu = m \cdot \cos\theta - n \cdot \sin\theta$$
$$\nu = m \cdot \sin\theta + n \cdot \cos\theta \qquad (1)$$



(a) Dot array of screen



(b) Spectrums in the spatial frequency domain

Figure 1. Relationship between dot array in the image domain and and corresponding spectrums in the spatial frequency domain. (a) Dot array of screen, (b) Spectrums in the spatial frequency domain

The frequency components of the overlapped images can be solved by using the convolution of two frequency distribution patterns of respective images.

When $\theta = \pm 15°$ and a screen ruling of N = 1/d (d: screen pitch), then the frequency components of the overlapped image are shown in Fig. 2.

The noteworthy point of this figure is that the frequency components appear in the same position of the frequency components of 45 degrees screen having the screen ruling of N=1/d. The coordinates are shown by equation (2).

$$\mu_k = \pm k(\cos 15° - \sin 15°)$$
$$\nu_l = \pm l(\cos 15° - \sin 15°) \qquad (2)$$

Where k, l : 0,±1, ±2, ±3 ..... and (k+ l)=even number

In the next case that the screen having the screen angle of 45 degrees and the screen ruling of N=1/d is superimposed on the two overlapped halftones of ±15 degrees and N=1/d, the consequent frequency spectrums appears like as shown in Fig. 3.

The moire frequency components by which rosette patterns are created can be understood from Fig. 2 and Fig. 3. That is, the low spatial frequency components around the origin in the frequency domain correspond to the rosette pattern in the image domain.

The frequency components around the origin have positions on both circles of $r_1 = 0.5176/d$ and $r_2 = 0.2679/d$. This circular pattern is named "the Rosette Pattern in the Frequency domain (**RPF**)" in this paper. It is needless to say that the circular RPF corresponds to the Rosette Pattern in the Image domain (**RPI**).

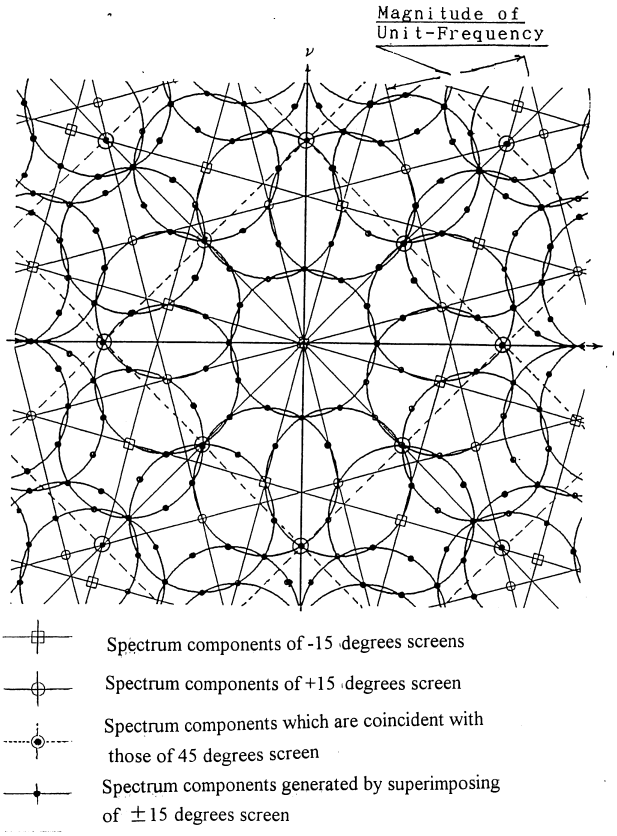


Figure 2. Spatial frequency components observed when the +15 degree screen is superimposed on the -15 degree screen.

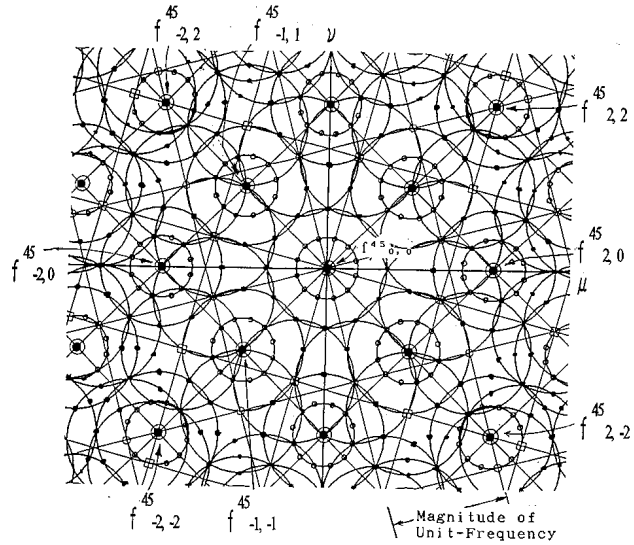


Figure 3. Rosette Pattern in spatial Frequency domain (RPF) when the most adequate 45 degrees screen is superimposed on the overlapped halftone of ± 15 degrees screens.

3.0 ±15 Degrees RT Screens and 45 Degrees RT Screen

3.1 Approximation of tan(±15°) by Rational Tangent

Rational tangent values which satisfy the conditions of 1/3>tan15°>1/4 are as follows;

1/4	rational tangent values p/q				1/3
2/8		2/7			2/6
3/12	3/11		3/10		3/9
4/16	4/15,	4/14	4/13		4/12
5/20	5/19,	5/18,	5/17	5/16	5/15

If the rational tangent screening of 15 degrees is required, p/q values sandwiched in between 1/3~1/4 must be used. We investigated spectrums of superimposed halftones having rational tangent of ±p/q and found that spectrums appear on the coordinates of (μ_k, ν_l) in every case.

$$\mu_k = \pm k(\cos\theta - \sin\theta) -$$
$$\nu_l = \pm l(\cos\theta - \sin\theta) \qquad (3)$$

Where k, l : 0,±1, ±2, ±3 ..... and (k + l) = even value

The coordinates of those spectrum components coincide with those of 45 degrees screen.

3.2 Generation of Rosette Pattern and its Shape

When superimposing three halftones having screen angles of ±15 degrees and 45 degrees for which the spectrums are given by equation (3), then a rosette pattern appears on the halftone image. It is able to recognize easilly that a specific RPI depends on a specific shape of RPF. The precise shape and spectrum coordinates of the RPF are shown in Fig. 4 and Table 1.

The spectrum coordinates are calculated by using Table 1 when θ is nearly equal to 15 degrees and tan θ is

expressed as a rational number. The shape of the spectrum is shown by Fig. 5 in detail. In the various shapes of spatial frequency spectrums, we can see the circular shapes of the inner ring only when p/q= 3/11, 4/15 and 5/19.

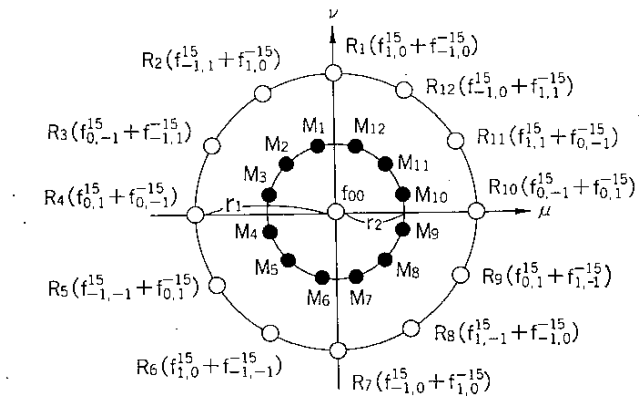


Figure 4. Precise shape of RPF around the origin when 3 screens having screen angle of ± θ degrees(θ is nearly equal to 15 degrees) and 45 degrees are superimposed .

Table 1. Low frequency spectrum components (rosette pattern in the frequency domain) observed around the origin when three halftone screen having screen angles of θ (nearly equal to ±15 degrees) and 45 degrees are superimposed.

(a) Coordinates of spectrum components on the outer ring

R1 = (0, 2sinθ) R2 = (–sinθ, cosθ–2sinθ)
R3 = (–cosθ+2sinθ, sinθ) R4 = (–2sinθ, 0)
R5 = (–cosθ+2sinθ, –sinθ) R6 = (–sinθ, cosθ+2sinθ)
R7 = (0, –2sinθ) R8= (sinθ, cosθ+2sinθ)
R9 = (cosθ–2sinθ, –sinθ) R10=(2sinθ, 0)
R11 =(cosθ–2sinθ, sinθ) R12=(sinθ, cosθ–2sinθ)

(b) Coordinates of spectrum components on the inner ring

M1 =(cosθ–4 sinθ, sinθ)
M2=(–cosθ+3 sinθ, cosθ–3 sinθ)
M3 =(–sinθ, –cosθ+ 4 sinθ)
M4 =(–sinθ, cosθ–4 sinθ)
M5 =(–cosθ+3 sinθ,–cosθ+3 sinθ)
M6 =(cosθ–4 sinθ,–sinθ)
M7 =(–cosθ+4 sinθ,–sinθ)
M8 =(cosθ–3 sinθ,–cosθ+3 sinθ)
M9 =(sinθ, cosθ–4 sinθ)
M10 =(sinθ, –cosθ+4 sinθ)
M11 =(cosθ–3 sinθ,cosθ–3 sinθ)
M12 =(–cosθ+ 4 sinθ, sinθ)

Consequently if the rational tangent value of 3/11, 4/15 or 5/19 is used to make the 15° screen then the shape of rosette in the image domain becomes circular. However, the next issue is how to convert the rational tangent value of p/q to the dot alignment in the image domain.

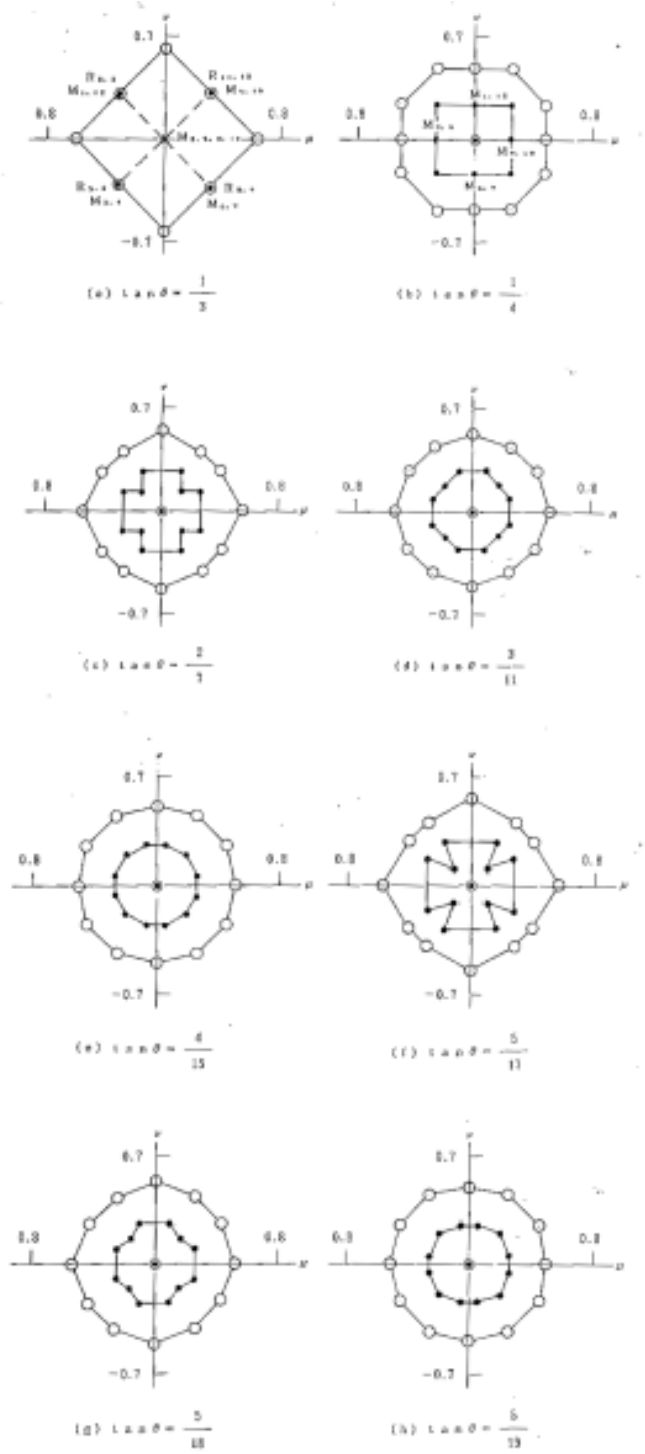
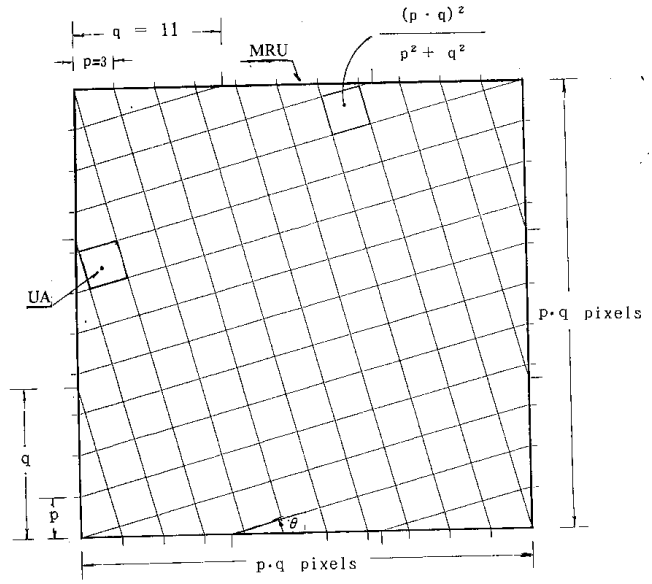


Figure 5. Various shapes of spatial frequency spectrums which appear on around the origin when ±15 degrees screens having rational tangent values and the corresponding 45 degrees screen to be paired with the ±15 degrees screens are superimposed. (Unit: reciprocal of screen pitch of 15 degrees screen)

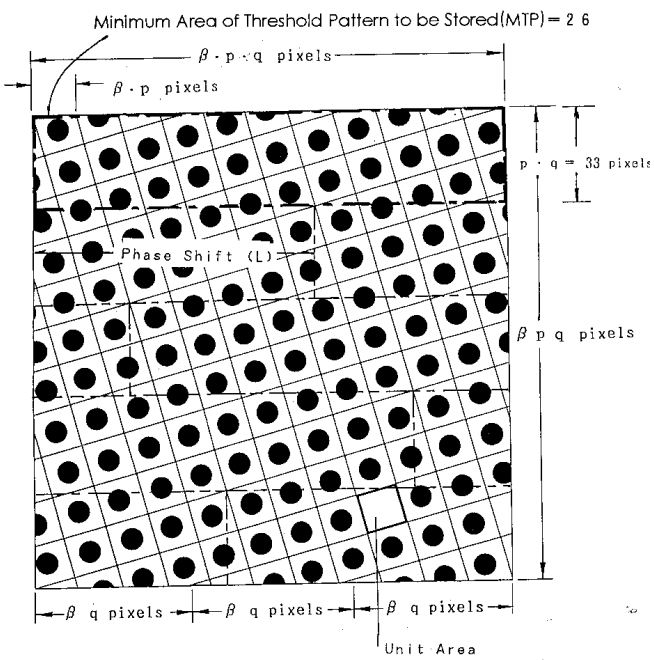
4.0 Construction of Digital Screen Set by The Multi-unit Area Design Method

4.1 Minimum Repetition Unit and Unit Area

In this Section, how to design the dot disposition pattern is explained. As an example, p/q=3/11 is used since this value is practical.



(a) Relationship between MRU and UAs included in it.



(b) Multi-unit area construction of RU to satisfy the tonal value requirement. RU is expressed by several MTPs and phase shift between adjacent MTPs.

Figure 6. Construction of 15-degrees halftone screen having rational tangent of 3/11.

If a square area of 33 pixels × 33 pixels is imagined and both horizontal and vertical sides of this area are divided by three and eleven, then a alignment of small square areas shown in Fig. 6 can be obtained. The alignment shown in Fig. 6 has the following specifications;

1. This area is divided into small squares and parts of square. The area of a small square (named Unit Area: **UA**) includes $(33 \times 33) - (3^2 + 11^2)$ pixels.
2. The square area of 33 pixels × 33 pixels is a minimum size tiling unit and can be put side by side in four directions continuously. So the square area of 33 pixels × 33 pixels including 130 small square areas is called the Minimum Repetition Unit (**MRU**) in this paper.
3. The tangent value of the UA alignment is equal to 3/11 and pitch of the UA alignment **d** is equal to $(3 \times 11) / \cdot (3^2 + 11^2)$ (pixels).

When the specifications written above are extended for the case of tanθ= p/q, then the following results is easily obtained.

number of pixels in MRU .....$(p \times q^2)$ pixels
number of UA in MRU .....$(p^2 + q^2)$ (4)
number of pixels in UA .....$(p \cdot q)^2 / (p^2 + q^2)$ (pixels)

4.2 Representable Tonal Values

When the halftone image designed by this method is applied to commercial use, more than 32 tonal values for low end printing machine and more than 200 tone values for graphic arts use are required. For this purpose of adjustment the scaling factor β is adopted so as to satisfy equation (5) as shown in Fig. 6(b). The area for which the size is β^2 times of MRU is named Repetition Unit (**RU**) and is the basic area to assign threshold values for every UA included in RU.

$$(\beta \cdot p \cdot q)^2 / (p^2 + q^2) \geq \text{Tonal values required} \qquad (5)$$

If β is applied, UA sizes $(\beta \cdot p \cdot q)^2 / (p^2 + q^2)$ pixels.

For example, when p=3 and q=11, then a size of UA is 8.3769 pixels. If the tonal value of more than 200 is required, then β=5 and the tonal value(= average UA size) becomes 212.7.

4.3 Division to Minimum Area of Threshold Matrix Pattern to be Stored in the Memory

The screen information is normally stored in a digital memory that is accessed by the longitudinal and vertical addresses on the image, so how to make small the repeating area to be stored is important. In the case of β =5 p=3 and q=11, the RU can be divided into 5 parts as shown in Fig. 6 (b). In general, the number of rectangular repeating areas is equal to the least common multiple of β and the number of UAs in RU, and the each repeating area named the Minimum area of Threshold matrix Pattern to be stored in the memory (**MTP**) consists of the same threshold value alignment.[4]

However, there are phase difference of **L**(pixels) between adjacent MTP and the **L** value can be calculated from equation (6) by checking the combination of u and v values.

Table 2 Various parameters and values for proposed screen sets by which the traditional circular rosettes are produced.

Various parameters of θ = 15° screen	Rational tangent value (p/q)								
	3/11			4/15			5/19		
Number of pixels in MRU: $(p \cdot q)^2$	$(3 \times 1)^2$ = 1089 pixels			$(4 \times 15)^2$ = 3600 pixels			$(5 \times 19)^2$ = 9025		
Number of UA in MRU: $2(p^2 + q^2)$	130			241			386		
Number of pixels contained by UA in MRU	8,3769 pixels			14,9378 pixels			23,3808 pixels		
Scaling factors	2	3	5	2	3	4	1	2	3
Tonal value capable to represent	33.5	75.4	209.4	59.8	134.4	239.0	23.4	93.5	210.4
Number of pixels in MTP	2,176	9,801	5,445	14,400	32,400	57,600	9,025	18,050	81,225
Phase Shift value L	33	0	99	0	0	0	0	95	0
Various parameters of θ = 45° screen									
Diagonal length of UA in MRU: (p•q)/(q-p)	33/(11-3)=4.125 (pixels)			60/(15-4)=5.4545 (pixels)			95/(19-5)=6.7857 (pixels)		
Number of UA in MRU: $2(p + q)^2$	128			242			392		
Number of pixels contained by UA	8,5078 pixels			14,760 pixels			23,0230 pixels		
Scaling factors	2	3	5	2	3	4	1	2	3
Tonal value capable to represent	34.0	76.6	212.7	59.5	133.9	238.0	23.0	92.1	207.2
Number of pixels in MTP (pixels)	1,089	9,801	27,225	7,200	16,200	28,800	9,025	9,025	81,225

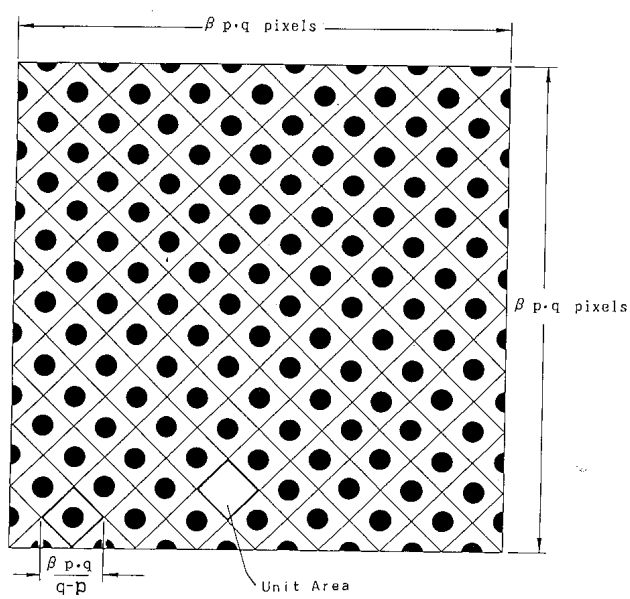


Figure 7. Construction of 45 degrees screen combined with ±15 degrees screens ($\tan^{-1}\theta$=3/11). The Repetition Unit area includes 128 UAs. Since 5 (value of β) and 128 (number of UAs) have no common divisor, the size of the MTP is equal to the size of the RU.

$$L = p^2 + v \cdot \beta \cdot p = -q^2 + u \cdot \beta \cdot q$$
$$(p^2 + q^2)/\beta = u \cdot q - v \cdot p \qquad (6)$$

where u and v are integers.

If tanθ=3/11 and β=5 in equation (6) then **L** = 99 (pixels), and for β= 2, **L**= 33 (pixels).

4.4 Minimum Repetition Unit at 45 Degrees

The 45 degrees screen which is combined with ±15 degrees screens having RT of ±p/q can be realized by the same way as for the 15 degrees screen. The length of horizontal or vertical side of the MRU is equal to (p•q).

Consequently the RU corresponding to Fig. 6(b) becomes the UA alignment shown in Fig. 7. A diagonal length of UA is calculated by equation (7).

$$\text{Diagonal length} = \text{unit length}/(\cos\theta - \sin\theta) = (p \cdot q)/(q - p) \qquad (7)$$

Since the number of pixles of both horizontal and vertical sides should be an integer, the size of square area is same as $(p \cdot q)^2$ (pixels). If p/q = 3/11, then integral number of UAs in MRU of 45 degrees screen becomes 2•(q-p) =128.

4.5 Screen Set Example

Table 2 shows some results of the screen set design for which moire and rosette behavior are already evaluated.

If the tonal value of more than 200 steps, resolution of almost 175 dpi and circular rosettes are required, then the following parameters in Table 2 satisfy the requirements.

CASE 1. tan θ = 4/15 and β =4 for 15 degree screen
CASE 2. tan θ = 3/11 and β =5 for 15 degree screen
CASE 3. tan θ = 5/19 and β = 3 for 15 degree screen

When a recording resolution of 2700 dpi is assumed in CASE 1, screen set described below is obtained.

	Screen angle	Screen ruling	Tonal value
Separation 1	+15 degree	174.65 LPI	239.0
Separation 2	–15 degree	174.65 LPI	239.0
Separation 3	45 degree	175.00 LPI	238.0

5.0 How to Produce the Threshold Matrix

Once the MRU is established, the halftoning system must apply it on a pixel-by-pixel basis. To accomplish this, we must develop the threshold mapping method threshold for the screen angle of $\theta = \tan^{-1}(p/q)$. The design procedure is as follows;

1) Definition of Standard Pattern Function

At first, Standard Pattern (SP) which area is $-1 \cdot x < 1$, $-1 \cdot y < 1$ and the value in any position can be calculated by equation (8) named Standard Pattern Function (SPF) **T** is defined.

$$\mathbf{T} = k\cdot(|x| + |y|)/2 \qquad (8)$$

k • tonal values

The value of **T** corresponds to the threshold value at a position (x,y).

2) Decision of the size of RU area $(\beta\cdot p\cdot q)^2$ considering the required tonal values

3) Decision of the space required to map threshold values on it

As illustrated in Fig. 8, the area in which the margin to paste up is added to the RU is defined for mapping of threshold values. The width of margin to paste up ($\alpha\cdot\beta$) should fulfill equation (9).

$$\alpha \geq \min(p, q) \qquad (9)$$

4) Repeating of SP areas is performed as shown in Fig. 8.

5) Mapping of threshold values

After setting the side length of the SP area equal to that of UA, SP area array is rotated and superimposed on the area of the RU added by margin to paste up. The values of SPF after rotated are assigned for every pixels in squares included in the RU added by margin to paste up. Zero is assigned to each pixel of the squares, the full shape of which are excluded in the RU plus margin area.

6) Optimization of Threshold Values

After assignment of threshold values, the proportional adjustment between the tonal values and the quantized signal levels and rounding are performed. For instance, the maximum tonal value should be changed equal to the maximum signal value of 255. Moreover, the threshold value alignment is checked so as not to have the same values in the same UA or in adjacent pixels or not to appear the declination of values in a specific direction, and after that, the threshold matrix which size is equal to $(\beta\cdot p\cdot q)^2$ pixels is completed by cutting off the margin area to paste up.

6.0 Results

Figure 9 shows the printed sample using halftones produced by this method. One can observe the rosette pattern predicted theoretically. As for the shape of rosettes, there are two types. One is the dot-centered rosette and the other is the clear-centered rosette.

This difference of the rosette shape is caused only by the difference of dot alignment of the superimposed halftones. When the 45 degrees screen is shifted about a half of screen pitch, then the shape of rosette pattern changes from the dot-centered to the clear-centered and vice versa. Both type of rosette pattern are depicted in Fig. 10.

7.0 Conclusion and Remarks

A new design method for screen set is proposed and the spectrum distribution of superimposed halftones is analyzed as the basis of this method.

In the way of study, it becomes clear that the design method includes the Hell's well-known screening method as a special case of p/q = 1/3.[5] The investigation in detail makes clear the relationship between rational tangent values and rosette pattern shape in the spatial frequency domain.

The Minimum Repetition Unit is the fundamental concept of this paper and is understood as the conversion of spectrum distribution of screens having rational tangent values.

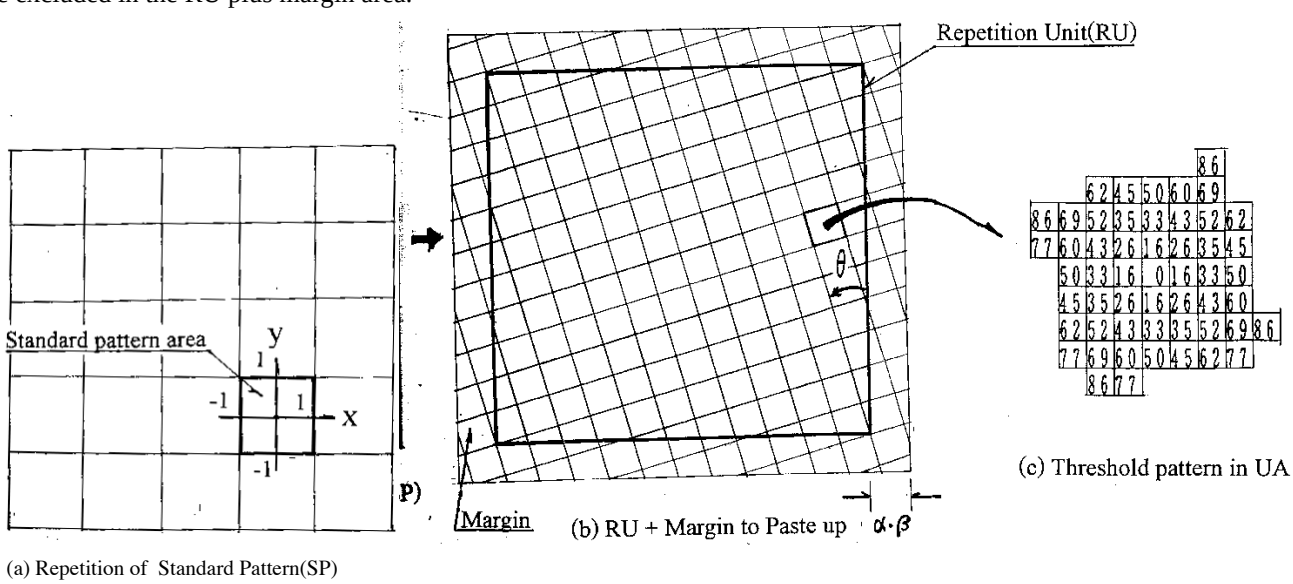
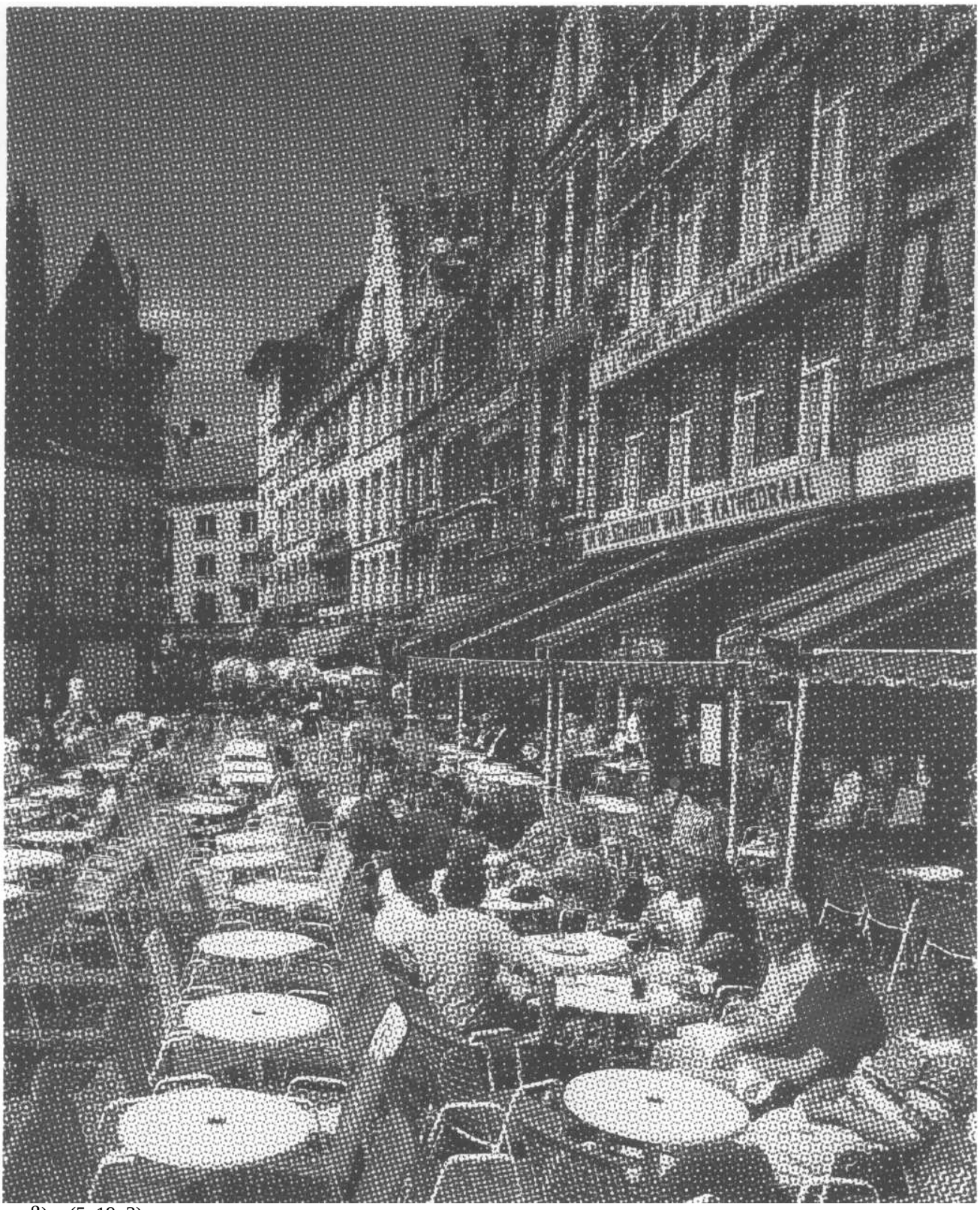
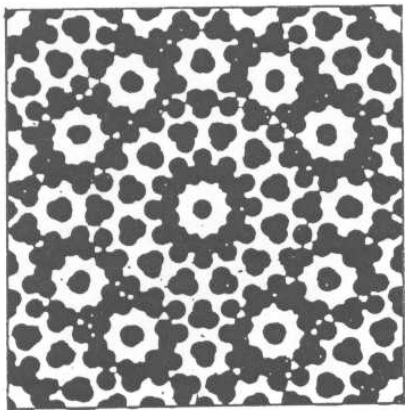


Figure 8. UA alignment and area to be considered for mapping of threshold values. Threshold values in UAs aligned in RU are calculated by rotating of SP area array.



(p, q, β) = (5, 19, 3)

Figure 9. Printed sample of SCID in which circular rosettes can be observed. Recording condition is as follows; tan θ = p/q =5/19 , β =3, Cyan:-15 degree, Magenta: +15 degree; Black and Yellow: 45 degree, Resolution: 300 dpi



(a) Dot-centered Rosettes

(b) Clear-centered Rosettes

Figure 10. (a) shows Dot-centered Rosettes and (b) shows Clear-centered Rosettes. Both are generated by the method proposed in this paper. Difference of these rosettes comes from the phase shift of superimposed 45 degree screen.

We adopted the scaling factor β to adjust the tonal value to the required value. In this paper, β is assumed integer. However, β could be various rational values and the integral study of it will be covered in a future paper. The Repetition Unit size of screens designed by the Multi-unit Area Method is usually larger than that of the Adobe Accurate Screen designed in the image domain. So we ought to develop the mapping method of threshold pattern.

Our conclusion is that we propose the new design method of screen set having the rational tangent values and the quality of the prints produced by the realized screen set is good enough for commercial use. And finally, the control method of rosette pattern seems to be an interesting issue to be resolved.

Reference

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